Distributed control of second-order nonlinear time-delayed multiagent systems with disturbance using neural networks

Hongwen Ma, *Student Member, IEEE*,
Supervisor: Derong Liu, *Fellow, IEEE* and Zhong-Ping Jiang, *Fellow, IEEE*

***Abstract*—In this paper, a class of second-order nonlinear time-delayed multiagent systems with disturbance is investigated. In order to improve the adaptivity, neural networks are used to learn the unknown dynamics. Then, by utilizing Lyapunov-Krasovskii functional, time delays can be eliminated. Moreover, a robustifying term is introduced to constrain external disturbance. With divide-and-conquer idea, the distributed controller is divided into five different parts to make the multiagent systems reach consensus. To circumvent singularity induced by the time-delay elimination part, a σ-function is developed. Finally, the simulation results demonstrate the validity of the distributed controller.**

***Index Terms*—Distributed control; Disturbance; Neural networks; Second order; Time-delayed multiagent systems**

I. Introduction

Distributed control is an important technique in multiagent systems. It can be traced back to Boid model [1] and Vicsek model [2], which are derived from natural phenomena. Variety of problems investigated include optimal control problems [3]–[5], output-based control problems [6]–[8], event-triggered control problems [9], [10] and time-delayed control problems [11], [12]. For more details, please refer to the survey papers [13]–[17] and the references therein. In [18], a decentralized adaptive control with neural networks (NNs) was established for multiagent systems with unknown dynamics. In [12], a class of first-order nonlinear time-delayed multiagent systems with external noises is studied. In [11], a Lyapunov-Krasovskii functional and Young's inequality were used for the consensus of time-delayed multiagent systems. Thus, it is of great significance to investigate how to apply the distributed control technique to second-order nonlinear time-delayed multiagent systems.

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Hongmen Ma is with The State Key Laboratory of Management and Control for Complex Systems, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, China (e-mail: mahongwen2012@ia.ac.cn). Derong Liu is with the School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing 100083, China. Zhong-Ping Jiang is with Polytechnic School of Engineering, New York University, New York, USA.

The technique of NNs is a powerful tool for learning the unknown dynamics [19]. In [20], adaptive neural control was introduced to solve the uncertain MIMO nonlinear systems. In [21], an adaptive neural control protocol was utilized for a class of strict-feedback nonlinear systems with unknown time delays. We utilize the technique of Lyapunov-Krasovskii functional from [21] and [11] to eliminate the negative effects of time delays. However, this technique will induce singularities in the distributed controller and a σ-function is established to deal with it.

To the best of our knowledge, it is the first time to investigate second-order time-delayed nonlinear multiagent systems with the developed σ-function. A reference signal which can reduce the difficulty of achieving consensus is also applied. Furthermore, by using the property of hyperbolic tangent function, a robustifying term is utilized to constrain the disturbance.

The rest of this paper is organized as follows. Preliminaries for graph theory and radial basis function neural networks (RBFNNs) are given in Section II. Main results are given in Section III. Simulation example is conducted to demonstrate the effectiveness of the developed method in Section IV. Conclusion is given in Section V.

Notations: $(\cdot)^{\mathsf{T}}$ denotes the transpose of a given matrix. $\mathrm{tr}\,(\cdot)$ is the trace of a given square matrix. $\|\cdot\|$ is the Frobenius norm or Euclidian norm. $\otimes$ stands for the Kronecker product. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the smallest nonzero eigenvalue and the largest eigenvalue of a given real symmetric matrix, respectively. $\mathrm{diag}(\cdot)$ represents a diagonal matrix.

II. Preliminaries

A. Graph Theory

A triplet $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is called a graph if $\mathcal{V} = \{1, 2, \ldots, N\}$ is the set of nodes, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges, and $\mathcal{A} = (\mathcal{A}_{ij}) \in \mathbb{R}^{N\times N}$ is the adjacency matrix of $\mathcal{G}$. Denote $\mathcal{A}_{ij}$ as the element of the ith row and jth column of the matrix $\mathcal{A}$. The ith node represents the ith agent, and an ordered pair $(i, j) \in \mathcal{E}$ means that agent i can directly transfer its information to agent j. No self-loop will be considered.

Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is given as follows:

$$\mathcal{L}_{ij}=\begin{cases}\sum\limits_{k\in\mathcal{N}_i}\mathcal{A}_{ik}, & \text{if } i=j;\\ -\mathcal{A}_{ij}, & \text{if } i\neq j.\end{cases} \tag{1}$$

B. Radial Basis Function Neural Networks

In practice, we usually employ a neural network as the function approximator to model an unknown function. RBFNN is a potential candidate for approximating the unknown dynamics of the multiagent systems. A continuous unknown nonlinear function vector $h(x)=[h_1(x),h_2(x),\ldots,h_m(x)]^{\mathsf{T}}\colon\mathbb{R}^m\to\mathbb{R}^m$ can be approximated by RBFNNs:

$$h(x)=W^{\mathsf{T}}\Phi(x), \tag{2}$$

where $x=[x_1,x_2,\ldots,x_m]^{\mathsf{T}}\in\mathbb{R}^m$ is the input vector, $W\in\mathbb{R}^{p\times m}$ is the weight matrix and p represents the number of neurons. $\Phi(x)=[\varphi_1(x),\varphi_2(x),\ldots,\varphi_p(x)]^{\mathsf{T}}$ is the activation function vector and

$$\varphi_i(x)=\exp\left[\frac{-(x-\mu_i)^{\mathsf{T}}(x-\mu_i)}{\delta_i^2}\right],\ i=1,2,\ldots,p, \tag{3}$$

where $\mu_i=[\mu_{i1},\mu_{i2},\ldots,\mu_{im}]^{\mathsf{T}}$ is the center of receptive field and δ_i is the width of Gaussian function. For a given positive constant θ_N, there exists an ideal weight matrix W^* such that

$$h(x)=W^{*\mathsf{T}}\Phi(x)+\theta, \tag{4}$$

where $\theta\in\mathbb{R}^m$ is the approximation error with $\|\theta\|<\theta_N$. However, it is difficult to obtain W^* in physical implementations. Therefore, we denote $\hat{W}$ as the estimation of the ideal weight matrix W^*. The estimation of $h(x)$ can be written as

$$\hat{h}(x)=\hat{W}^{\mathsf{T}}\Phi(x), \tag{5}$$

where $\hat{W}$ can be updated online. The online updating algorithm will be provided in Section III.

III. Main Results

We discuss the second-order multi-agent system and it can be described as follows:

$$\begin{aligned}\ddot{x}_i(t)&=f_i(x_i(t),\dot{x}_i(t))+g_i(\dot{x}_i(t-\tau_i))+u_i(t)+\xi_i(t),\\ &\quad i=1,2,\ldots,N,\end{aligned} \tag{6}$$

where $x_i(\cdot)\in\mathbb{R}^m$ is the state vector, τ_i and $\xi_i(\cdot)\in\mathbb{R}^m$ represent the unknown time delay and disturbance, respectively. $u_i(\cdot)\in\mathbb{R}^m$ is the control vector, $f_i(\cdot)\colon\mathbb{R}^m\to\mathbb{R}^m$ and $g_i(\cdot)\colon\mathbb{R}^m\to\mathbb{R}^m$ are continuous but unknown nonlinear vector functions. Here we assume $\|\xi_i\|<\alpha_i$ where $\alpha_i>0$. For simplicity, in the sequel we will ignore time expression t in case there is no confusion.

Our aim is to design a distributed controller which can make the nonlinear time-delayed multiagent systems reach consensus. The distributed controller is divided into five parts and they are linear feedback term, neural network term, time-delay elimination term, robustifying term and second-order information term. Before proceeding, we introduce a Lyapunov-Krasovskii functional as follows:

$$L_Q(t)=\frac{1}{2}\sum_{i=1}^{N}\int_{t-\tau_i}^{t}Q_i(\dot{x}_i(\zeta))\mathrm{d}\zeta, \tag{7}$$

where $Q_i(\dot{x}_i(\zeta))=\phi_i^2(\dot{x}_i(\zeta))$ and $\phi_i(\cdot)$ is a scalar function satisfying $\phi_i(x_i)\geq\|g_i(x_i)\|$. The time derivative of $L_Q(t)$ is

$$\dot{L}_Q(t)=\frac{1}{2}\sum_{i=1}^{N}\left(\phi_i^2(\dot{x}_i(t))-\phi_i^2(\dot{x}_i(t-\tau_i))\right). \tag{8}$$

Let $z_{i1}=x_i$ and $z_{i2}=\dot{x}_i$. Then, (6) can be rewritten in the following form:

$$\begin{cases}\dot{z}_{i1}=z_{i2},\ i=1,2,\ldots,N,\\ \dot{z}_{i2}=f_i(z_{i1},z_{i2})+g_i(z_{i2}(t-\tau_i))+u_i+\xi_i.\end{cases} \tag{9}$$

In the sequel, for convenient analysis, we will ignore the declaration that $i=1,2,\ldots,N$ and concentrate on agent i. Suppose that

$$z_{i2d}=-k_i\sum_{j\in\mathcal{N}_i}A_{ij}(z_{i1}-z_{j1}), \tag{10}$$

and we can obtain an error signal between the real state z_{i2} and the virtual state z_{i2d}, i.e., $v_{ei}=z_{i2}-z_{i2d}$. Consequently, the time derivative of v_{ei} is

$$\begin{aligned}\dot{v}_{ei}&=\dot{z}_{i2}-\dot{z}_{i2d}\\ &=f_i(z_{i1},z_{i2})+g_i(z_{i2}(t-\tau_i))+u_i\\ &\quad+\xi_i+k_i\sum_{j\in\mathcal{N}_i}A_{ij}(z_{i2}-z_{j2}).\end{aligned} \tag{11}$$

We utilize RBFNNs to approximate $f_i(z_{i1},z_{i2})$. The distributed controller is designed as follows:

$$\begin{aligned}u_i=&-\rho_i(t)v_{ei}-\hat{W}_i^{\mathsf{T}}\Phi_i(z_i)-\frac{1}{2}\frac{v_{ei}}{\|v_{ei}\|^2+\sigma(v_{ei})}\phi_i^2(z_{i2})\\ &-\gamma_i\tanh\left(\frac{\kappa_i\gamma_i v_{ei}}{\epsilon_i}\right)-k_i\sum_{j\in\mathcal{N}_i}\mathcal{A}_{ij}(z_{i2}-z_{j2}),\end{aligned} \tag{12}$$

where

$$\rho_i(t)=k_{i0}+\frac{1}{2}+\frac{1}{2\omega_i}\left(1+\frac{1}{\|v_{ei}\|^2+\sigma(v_{ei})}\hbar_i\right), \tag{13}$$

$$\begin{aligned}\hbar_i&=\int_{t-\tau_{\max}}^{t}Q_i(z_{i2}(\zeta))\mathrm{d}\zeta+\omega_i\|z_{i2}\|^2\\ &\quad+(\omega_i+\lambda_{\max}(M))\|z_{ei}\|^2,\\ z_{ei}&=\sum_{j\in\mathcal{N}_i}\mathcal{A}_{ij}(z_{i1}-z_{j1}),\\ z_i&=[z_{i1}^{\mathsf{T}},z_{i2}^{\mathsf{T}}]^{\mathsf{T}},\\ \sigma(v_{ei})&=\begin{cases}1, & \text{if } \|v_{ei}\|=0,\\ 0, & \text{if } \|v_{ei}\|\neq 0.\end{cases}\end{aligned}$$

Furthermore, $\omega_i>0$, $\tau_{\max}>\tau_i>0$ and M is defined in (19). Next, we discuss the structure of the distributed controller.

1) The linear feedback term $-\rho_i(t)v_{ei}$ contains the information used by agent i to guide its direction towards consensus so that z_{i2} can track z_{i2d}. If consensus can be reached, then $-\rho_i(t)v_{ei}$ has no impact on the multiagent system (6).

2) Neural network term $-\hat{W}_i^{\mathsf{T}}\Phi_i(z_i)$ is used to learn the characteristics of $f_i(z_i)$ online. $\hat{W}_i$ represents the estimation of RBFNN weight matrix of agent i. The adaptive updating algorithm is given as follows:

$$\dot{\hat{W}}_i = \begin{cases} a_i\Phi_i(z_i)v_{ei}^{\mathsf{T}}, \quad \text{if } \operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}\hat{W}_i\right) < W_i^{\max}, \quad \text{or} \\ \text{if } \operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}\hat{W}_i\right) = W_i^{\max} \text{ and } v_{ei}^{\mathsf{T}}\hat{W}_i^{\mathsf{T}}\Phi_i(z_i) < 0; \\ a_i\Phi_i(z_i)v_{ei}^{\mathsf{T}} - a_i\dfrac{v_{ei}^{\mathsf{T}}\hat{W}_i^{\mathsf{T}}\Phi_i(z_i)}{\operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}\hat{W}_i\right)}\hat{W}_i, \\ \text{if } \operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}\hat{W}_i\right) = W_i^{\max} \text{ and } v_{ei}^{\mathsf{T}}\hat{W}_i^{\mathsf{T}}\Phi_i(z_i) \geq 0, \end{cases} \tag{14}$$

where $a_i > 0$ and $W_i^{\max} > 0$. Suppose θ_i is the approximation error of the weight matrix of agent i satisfying $\|\theta_i\| < \theta_{N_i}$. It is noted that the initial value $\hat{W}_i(0)$ satisfies

$$\operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}(0)\hat{W}_i(0)\right) \leq W_i^{\max}. \tag{15}$$

Thus, we let $\hat{W}_i(0)$ be a zero matrix. Furthermore, according to Lemma 2 in [18], if the updating algorithm is expressed as (14), then $\operatorname{tr}\left(\hat{W}_i^{\mathsf{T}}(t)\hat{W}_i(t)\right) \leq W_i^{\max}, \forall t \geq 0$.

3) $-\dfrac{1}{2}\dfrac{v_{ei}}{\|v_{ei}\|^2 + \sigma(v_{ei})}\phi_i^2(z_{i2})$ is the time-delay elimination term which is introduced to eliminate the effects of time delays. $\|v_{ei}\| = 0$ will induce an infinite control beyond physical implementations. Thus, we should exclude zero case and σ-function is a wise choice for solving this problem.

4) $-\gamma_i \tanh\left(\dfrac{\kappa_i\gamma_i v_{ei}}{\epsilon_i}\right)$ is to constrain disturbance ξ_i and RBFNN approximation error θ_i, where $\kappa_i = 0.2785$ (more details can be found in [19]). Furthermore, γ_i is the robust gain satisfying

$$\gamma_i \geq \theta_{N_i} + \alpha_i \tag{16}$$

and $\epsilon_i > 0$. By virtue of Lemma 1 in [19], we can easily get the following inequalities:

$$\gamma_i\|v_{ei}\| + v_{ei}^{\mathsf{T}}\gamma_i \tanh\left(\frac{\kappa_i\gamma_i v_{ei}}{\epsilon_i}\right) \leq \epsilon_i. \tag{17}$$

5) $-k_i \sum\limits_{j\in\mathcal{N}_i} \mathcal{A}_{ij}(z_{i2} - z_{j2})$ is the second-order information term and it includes the information of velocities that agent i can obtain from its neighbors. If consensus is achieved, velocities are zeros and this term becomes zero.

Theorem 1: The second-order multiagent system is given in (6). If the distributed controller is given in (12) and the online updating algorithm of the weight matrix is expressed as (14), then the multiagent system (6) can achieve consensus.

Proof: We construct a Lyapunov function containing error signal v_{ei}, $i = 1, 2, \ldots, N$, as follows:

$$V(t) = V_{\hat{z}_1}(t) + L_Q(t) + \frac{1}{2}\sum_{i=1}^{N}\operatorname{tr}\left(\frac{1}{a_i}\tilde{W}_i^{\mathsf{T}}\tilde{W}_i\right) + \frac{1}{2}v_e^{\mathsf{T}}v_e \tag{18}$$

where $\tilde{W}_i = W_i^* - \hat{W}_i$, $V_{\hat{z}_1}(t) = \dfrac{1}{2}\hat{z}_1^{\mathsf{T}}(\mathcal{L}\otimes I_m)\hat{z}_1$, $\hat{z}_1 = [z_{11}^{\mathsf{T}}, z_{21}^{\mathsf{T}}, \ldots, z_{N1}^{\mathsf{T}}]^{\mathsf{T}}$, $L_Q(t) = \dfrac{1}{2}\sum\limits_{i=1}^{N}\int\limits_{t-\tau_i}^{t} Q_i(z_{i2}(\zeta))\mathrm{d}\zeta$ and $v_e = [v_{e1}^{\mathsf{T}}, v_{e2}^{\mathsf{T}}, \ldots, v_{eN}^{\mathsf{T}}]^{\mathsf{T}}$. Note that if $\|v_{ei}\| = 0$, then $z_{i2} = z_{i2d}$, i.e., $\dot{z}_{i1} = z_{i2d} = -k_i \sum\limits_{j\in\mathcal{N}_i} \mathcal{A}_{ij}(z_{i1} - z_{j1})$. It it obvious that this is a traditional distributed control for consensus. Therefore, in the sequel we will focus on the case where $\|v_{ei}\| \neq 0$. Then, we can infer that

$$\begin{aligned}
\frac{\mathrm{d}V(t)}{\mathrm{d}t} &= z_e^{\mathsf{T}}\dot{\hat{z}}_1 + \frac{1}{2}\sum_{i=1}^{N}\Big(\phi_i^2(z_{i2}(t)) - \phi_i^2(z_{i2}(t-\tau_i))\Big) \\
&\quad - \sum_{i=1}^{N}\operatorname{tr}\left(\frac{1}{a_i}\tilde{W}_i^{\mathsf{T}}\dot{\hat{W}}_i\right) + v_e^{\mathsf{T}}\dot{v}_e \\
&= z_e^{\mathsf{T}}\hat{z}_2 + \frac{1}{2}\sum_{i=1}^{N}\Big(\phi_i^2(z_{i2}(t)) - \phi_i^2(z_{i2}(t-\tau_i))\Big) \\
&\quad - \sum_{i=1}^{N}\operatorname{tr}\left(\frac{1}{a_i}\tilde{W}_i^{\mathsf{T}}\dot{\hat{W}}_i\right) + \sum_{i=1}^{N}v_{ei}^{\mathsf{T}}(u_i + \xi_i - \dot{z}_{i2d}) \\
&\quad + \sum_{i=1}^{N}v_{ei}^{\mathsf{T}}\Big(f_i(z_{i1}, z_{i2}) + g_i(z_{i2}(t-\tau_i))\Big),
\end{aligned}$$

where $\hat{z}_2 = [z_{12}^{\mathsf{T}}, z_{22}^{\mathsf{T}}, \ldots, z_{N2}^{\mathsf{T}}]^{\mathsf{T}}$ and $z_e = [z_{e1}^{\mathsf{T}}, z_{e2}^{\mathsf{T}}, \ldots, z_{eN}^{\mathsf{T}}]^{\mathsf{T}}$. The communication topology is connected, thus zero is an m-multiplicity eigenvalue of $\mathcal{L}\otimes I_m$ and T contains eigenvectors of $\mathcal{L}\otimes I_m$ corresponding to the eigenvalue matrix $\Lambda = \operatorname{diag}(0I_m, \lambda_2 I_m, \lambda_3 I_m, \ldots, \lambda_n I_m)$, where $TT^{\mathsf{T}} = T^{\mathsf{T}}T = I_{mN}$ and $T^{-1} = T^{\mathsf{T}}$. Hence,

$$\begin{aligned}
\hat{z}_1^{\mathsf{T}}(\mathcal{L}\otimes I_m)\hat{z}_1 &= \hat{z}_1^{\mathsf{T}}T^{\mathsf{T}}\Lambda T\hat{z}_1 \\
&= \hat{z}_1^{\mathsf{T}}T^{\mathsf{T}}\sqrt{\Lambda}\sqrt{\Lambda}T\hat{z}_1 \\
&= \hat{z}_1^{\mathsf{T}}T^{\mathsf{T}}\sqrt{\Lambda}\sqrt{\bar{\Lambda}}\sqrt{\bar{\Lambda}^{-1}}\sqrt{\bar{\Lambda}^{-1}}\sqrt{\bar{\Lambda}}\sqrt{\Lambda}T\hat{z}_1 \\
&= \hat{z}_1^{\mathsf{T}}T^{\mathsf{T}}\Lambda TT^{\mathsf{T}}\bar{\Lambda}^{-1}TT^{\mathsf{T}}\Lambda T\hat{z}_1 \\
&= \hat{z}_1^{\mathsf{T}}(\mathcal{L}\otimes I_m)^{\mathsf{T}}M(\mathcal{L}\otimes I_m)\hat{z}_1 \\
&= {z_e}^{\mathsf{T}}Mz_e,
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\sqrt{\Lambda} &= \operatorname{diag}(0I_m, \sqrt{\lambda_2}I_m, \sqrt{\lambda_3}I_m, \ldots, \sqrt{\lambda_n}I_m), \\
\bar{\Lambda} &= \operatorname{diag}(\lambda_2 I_m, \lambda_2 I_m, \lambda_3 I_m, \ldots, \lambda_n I_m), \\
\sqrt{\bar{\Lambda}} &= \operatorname{diag}(\sqrt{\lambda_2}I_m, \sqrt{\lambda_2}I_m, \sqrt{\lambda_3}I_m, \ldots, \sqrt{\lambda_n}I_m),
\end{aligned}$$

and $M = T^{\mathsf{T}}\bar{\Lambda}^{-1}T$. Then, we substitute (12) and (17) into

$\mathrm{d}V/\mathrm{d}t$ to obtain

$$\begin{aligned}\frac{\mathrm{d}V(t)}{\mathrm{d}t} \leq& \frac{1}{2}\sum_{i=1}^{N}(\|z_{ei}\|^2+\|z_{i2}\|^2)-\sum_{i=1}^{N}\mathrm{tr}\left(\frac{1}{a_i}\tilde{W}_i^{\mathsf{T}}\dot{\hat{W}}_i\right)\\&+\frac{1}{2}\sum_{i=1}^{N}\left(\phi_i^2(z_{i2}(t))-\phi_i^2(z_{i2}(t-\tau_i))\right)+\sum_{i=1}^{N}\epsilon_i\\&+\sum_{i=1}^{N}v_{ei}^{\mathsf{T}}\tilde{W}_i^{\mathsf{T}}\Phi_i(z_i)-\sum_{i=1}^{N}\left(\rho_i(t)-\frac{1}{2}\right)\|v_{ei}\|^2\\&+\frac{1}{2}\sum_{i=1}^{N}\left(\phi_i^2(z_{i2}(t-\tau_i))-\phi_i^2(z_{i2}(t))\right).\end{aligned}$$

With similar proof steps [12], we can obtain

$$\mathrm{tr}\left(\tilde{W}_i^{\mathsf{T}}\left(\frac{1}{a_i}\dot{\hat{W}}_i-\Phi_i(z_i)v_{ei}^{\mathsf{T}}\right)\right)\geq 0. \tag{20}$$

With $\tau_i<\tau_{\max}$, we have

$$\frac{1}{2}\sum_{i=1}^{N}\int_{t-\tau_i}^{t}Q_i(z_{i2}(\zeta))\mathrm{d}\zeta\leq\frac{1}{2}\sum_{i=1}^{N}\int_{t-\tau_{\max}}^{t}Q_i(z_{i2}(\zeta))\mathrm{d}\zeta.$$

Thus, with (13) and (19) we obtain

$$\begin{aligned}\frac{\mathrm{d}V(t)}{\mathrm{d}t}\leq&\sum_{i=1}^{N}\left(-k_{i0}\|v_{ei}\|^2-\frac{1}{2\omega_i}\|v_{ei}\|^2-\frac{\lambda_{\max}(M)}{2\omega_i}\|z_{ei}\|^2\right)\\&-\sum_{i=1}^{N}\frac{2W_i^{\max}}{\omega_s a_i}+\sum_{i=1}^{N}\frac{2W_i^{\max}}{\omega_s a_i}+\sum_{i=1}^{N}\epsilon_i\\&-\frac{1}{2\omega_i}\sum_{i=1}^{N}\int_{t-\tau_{\max}}^{t}Q_i(z_{i2}(\zeta))\mathrm{d}\zeta\\&-\mathrm{tr}\left(\tilde{W}_i^{\mathsf{T}}\left(\frac{1}{a_i}\dot{\hat{W}}_i-\Phi_i(z_i)v_{ei}^{\mathsf{T}}\right)\right)\\\leq&-\frac{1}{\omega_s}V_{\hat{z}_1}(t)-\frac{1}{\omega_s}L_Q(t)-\frac{1}{2\omega_s}v_e^{\mathsf{T}}v_e\\&-\frac{1}{2\omega_s}\sum_{i=1}^{N}\mathrm{tr}\left(\frac{1}{a_i}\tilde{W}_i^{\mathsf{T}}\tilde{W}_i\right)\\&+\sum_{i=1}^{N}\frac{2W_i^{\max}}{\omega_s a_i}+\theta_s\\\leq&-\frac{1}{\omega_s}V(t)+\sum_{i=1}^{N}\frac{2W_i^{\max}}{\omega_s a_i}+\theta_s,\end{aligned}$$

where $\omega_s=\max_{i\in\mathcal{V}}\omega_i$ and $\theta_s=\sum_{i=1}^{N}\epsilon_i$.

On the basis of Lemma 1 in [18], we obtain

$$V(t)\leq V(0)e^{-\frac{1}{\omega_s}t}+\nu_s\left(1-e^{-\frac{1}{\omega_s}t}\right), \tag{21}$$

where $\nu_s=\sum_{i=1}^{N}\frac{2W_i^{\max}}{a_i}+\omega_s\theta_s$. Since all the terms in (18) are nonnegative, as $t\to\infty$ we can obtain that $V_{\hat{z}_1}(t)\leq\nu_s$. That is, $\sum_{(j,i)\in\mathcal{E}}\mathcal{A}_{ij}(z_{i1}-z_{j1})^2\leq\nu_s$. By choosing the parameters $W_i^{\max}$, a_i, ω_i, ϵ_i and $\mathcal{A}_{ij}$ properly, we can eventually derive that

$$\|z_{i1}-z_{j1}\|\leq\sqrt{\frac{\nu_s}{\mathcal{A}_{ij}}},\ \forall(i,j)\in\mathcal{E}, \tag{22}$$

where $\sqrt{\nu_s/\mathcal{A}_{ij}}$ can be set small enough. Therefore, consensus can be achieved. ■

IV. Simulation Example

In this example, we utilize a multiple cooperative manipulator system to verify the validity of the distributed controller (12) in Section III. The two-link manipulator holds a component which is used to assemble the industrial product. The dynamics of the multi-manipulator system is described as follows:

$$M_i(q_i)\ddot{q}_i+V_i(q_i,\dot{q}_i)\dot{q}_i+G_i(q_i)+g_i(\dot{q}_i(t-\tau_i))+\xi_i(t)=\Gamma_i, \tag{23}$$

where $q_i=[q_{i1},q_{i2}]^{\mathsf{T}}\in\mathbb{R}^2$, $\dot{q}_i$ and $\ddot{q}_i$ are the position, velocity and acceleration vector of the ith manipulator, respectively. $M_i(q_i)\in\mathbb{R}^{2\times2}$ is the inertia matrix of manipulator i; $V_i(q_i,\dot{q}_i)\in\mathbb{R}^{2\times2}$ is the centripetal-Coriolis matrix of manipulator i; $G_i(q_i)\in\mathbb{R}^2$ is the gravitational vector of manipulator i and $\Gamma_i\in\mathbb{R}^2$ is the torque vector of manipulator i. We give the detail parameters of each manipulator as follows:

$$\begin{aligned}V_i(q_i,\dot{q}_i)=&\begin{bmatrix}V_{i11}&V_{i12}\\V_{i21}&V_{i22}\end{bmatrix},\\G_i(q_i)=&\begin{bmatrix}G_{i1}&G_{i2}\end{bmatrix},\\M_i=&I,\\V_{i11}=&-m_{i2}l_{i1}l_{i2}\sin(q_{i2})\dot{q}_{i2},\\V_{i12}=&-m_{i2}l_{i1}l_{i2}\sin(q_{i2})\dot{q}_{i2}-m_{i2}l_{i1}l_{i2}\sin(q_{i2})\dot{q}_{i1},\\V_{i21}=&m_{i2}l_{i1}l_{i2}\sin(q_{i2})\dot{q}_{i1},\\V_{i22}=&0,\\G_{i1}=&(m_{i1}+m_{i2})\tilde{g}l_{i1}\sin(q_{i1})+m_{i2}\tilde{g}l_{i2}\sin(q_{i1}+q_{i2}),\\G_{i2}=&m_{i2}\tilde{g}l_{i2}\sin(q_{i1}+q_{i2}).\end{aligned}$$

For simplicity, we set $M_i=I$. $g_i(\dot{q}_i(t-\tau_i))$ represents the friction force vector where

$$g_i(\dot{q}_i(t-\tau_i))=\begin{bmatrix}s_{i1}\dot{q}_{i1}(t-\tau_i)\cos(\dot{q}_{i2}(t-\tau_i))\\s_{i2}\dot{q}_{i2}(t-\tau_i)\sin(\dot{q}_{i1}(t-\tau_i))\end{bmatrix}. \tag{24}$$

We set the same parameters of all the six manipulators. $k_{i0}=15$, $\omega_i=30$, $W_i^{\max}=100$, $a_i=100$, $\gamma_i=2$, $\kappa_i=0.2785$ and $\epsilon_i=0.01$. The number of neurons for each RBFNN is 16 and $\delta_i^2=1.6$. μ_is are distributed uniformly among the range $[-3,3]\times[-3,3]$. The initial states of the multi-manipulator system are

$$\begin{bmatrix}q_1^{\mathsf{T}}(0)&\dot{q}_1^{\mathsf{T}}(0)\\q_2^{\mathsf{T}}(0)&\dot{q}_2^{\mathsf{T}}(0)\\q_3^{\mathsf{T}}(0)&\dot{q}_3^{\mathsf{T}}(0)\\q_4^{\mathsf{T}}(0)&\dot{q}_4^{\mathsf{T}}(0)\\q_5^{\mathsf{T}}(0)&\dot{q}_5^{\mathsf{T}}(0)\\q_6^{\mathsf{T}}(0)&\dot{q}_6^{\mathsf{T}}(0)\end{bmatrix}=\begin{bmatrix}[\pi/3,\pi/3]&[-1,1]\\[\pi/3,\pi/4]&[0.5,1]\\[\pi/5,-\pi/4]&[2,-1]\\[\pi/5,\pi/5]&[2,0.5]\\[-\pi/4,-\pi/6]&[3,-0.5]\\[-\pi/6,\pi/3]&[-1.5,0.5]\end{bmatrix}.$$

The Laplacian matrix is $\mathcal{L}$ given as follows:

$$\mathcal{L}=\begin{bmatrix} 0.8 & -0.5 & 0 & 0 & 0 & -0.3 \\ -0.5 & 0.7 & -0.2 & 0 & 0 & 0 \\ 0 & -0.2 & 0.9 & -0.7 & 0 & 0 \\ 0 & 0 & -0.7 & 1.8 & -1.1 & 0 \\ 0 & 0 & 0 & -1.1 & 3.1 & -2 \\ -0.3 & 0 & 0 & 0 & -2 & 2.3 \end{bmatrix}.$$

Other parameters are given in Tables I–III.

TABLE I
COEFFICIENT VALUES OF THE iTH MANIPULATOR

i	1	2	3	4	5	6
s_{i1}	0.9	1.2	−1.1	−0.7	0.6	0.3
s_{i2}	1.2	0.8	0.6	0.3	0.8	0.4

TABLE II
TIME DELAY OF THE iTH MANIPULATOR

i	1	2	3	4	5	6	$\tau_{\max}$
τ_i	0.1	0.05	0.15	0.08	0.18	0.1	0.2

TABLE III
PARAMETERS OF THE iTH MANIPULATOR

$\tilde{g}$	l_{i1}	l_{i2}	m_{i1}	m_{i2}
9.8 m/s^2	1.5 m	1 m	2 kg	1 kg

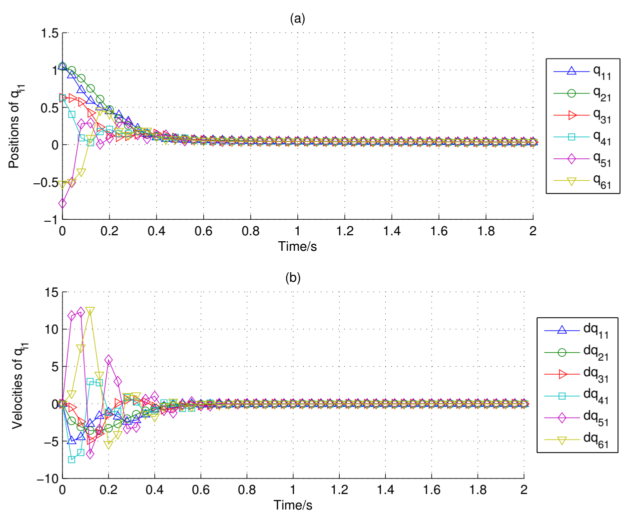

Fig. 1. Positions and velocities of link 1 of six manipulators. (a) Position trajectories of link 1 of six manipulators. (b) Velocity trajectories of link 1 of six manipulators.

From Fig. 1 and Fig. 2, we can infer that the multi-manipulator system (23) can reach the same position and velocity, where $\mathrm{d}q_{i1}$ and $\mathrm{d}q_{i2}$ represent the velocities of Link 1 and Link 2, respectively. In order to describe whether consensus has been achieved, we define the measurement of consensus error for each manipulator

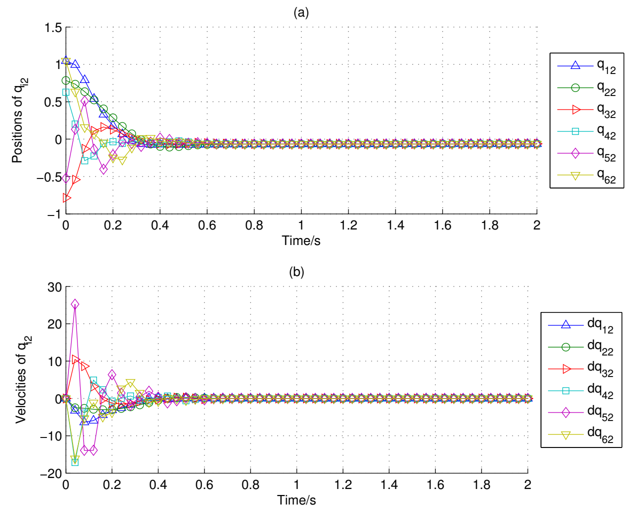

Fig. 2. Positions and velocities of link 2 of six manipulators. (a) Position trajectories of link 2 of six manipulators. (b) Velocity trajectories of link 2 of six manipulators.

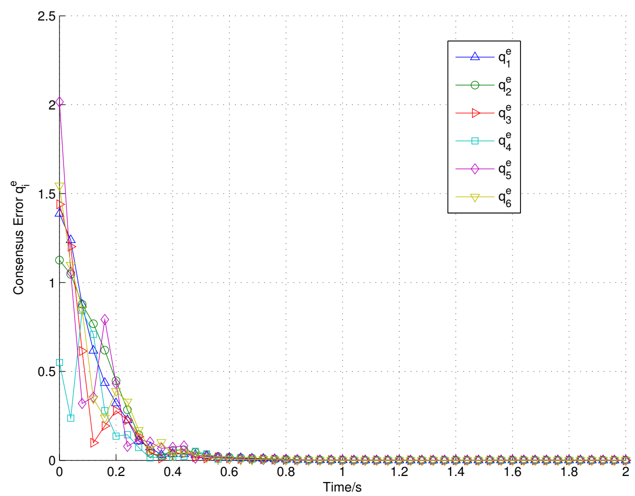

Fig. 3. Consensus error trajectories of six manipulators.

$$q_i^e=\left\|\sum_{j\in\mathcal{N}_i}\mathcal{A}_{ij}(q_i-q_j)\right\|,\ i=1,2,\ldots,6. \tag{25}$$

In Fig. 3, all the consensus errors approach zero. This further demonstrates the effectiveness of our design method.

V. CONCLUSION

A class of second-order nonlinear multiagent systems with disturbance and time delay are studied. The technique of Lyapunov-Krasovskii functional is utilized to eliminate time delay. However, singularities will be induced by this technique.

Thus, a σ-function is established to solve this problem. To deal with disturbance and the unknown nonlinear dynamics, a robustifying term and neural networks are introduced, respectively. Finally, the simulation example validates the effectiveness of the developed distributed controller.

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